TRINITY COLLEGE

MATHEMATICS SPECIALIST UNITS 1 AND 2
Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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Question 9 **(7 marks)**

(a)   Given that $\dfrac{18 \times 17 \times 16}{16 \times 15 \times 14 \times 13} = \dfrac{{}^{n}P_{r}}{{}^{m}P_{4}}$, determine the values of n, r and m. (3 marks)

(b)   Determine how many integers between 1 and 100 inclusive are divisible by 2, 3 or 7.
(4 marks)

See next page

Question 10 **(6 marks)**

(a)   In the circle shown below, minor arc PR subtends an angle of 120° at O, the centre of the circle, and the size of angle RPQ is 55°. Determine the size of angle POQ.   (2 marks)
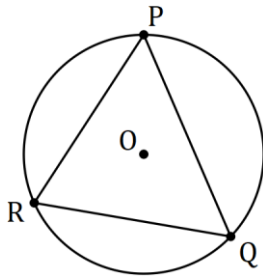
(b)   In the diagram below, AB is tangent to the circle with centre O at A, secant BD intersects the circle at C and D, and the sizes of angles AOC and COD are 72° and 104° respectively. Determine the size of angle ABC.   (4 marks)

See next page
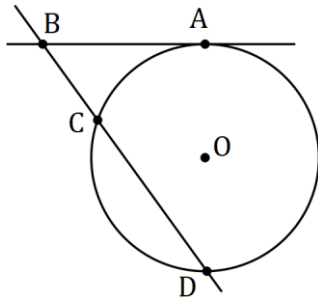
Question 11 **(8 marks)**

Two matrices are given by $P = \begin{bmatrix} 8 & 3 \\ -6 & 5 \end{bmatrix}$ and $Q = \begin{bmatrix} 5 & -3 \\ 6 & 8 \end{bmatrix}$.

(a) Determine PQ. (1 mark)

(b) Given that $Q^{-1} = kP$, determine the exact value of the constant k. (2 marks)

The system of equations $5a = 3b + 29$ and $6a + 8b + 87 = 0$ can be expressed as a matrix equation in the form $QX = R$.

(c) Determine matrices X and R. (2 marks)

(d) Express matrix X in terms of matrices P and R. (2 marks)

(e) Solve the system of equations. (1 mark)

Question 12 **(8 marks)**

(a)   Show how to express $0.\overline{23}$ as a rational number.   (2 marks)

(b)   Prove that the sum of any three consecutive integers is always a multiple of three.

(3 marks)

(c)   Prove by contradiction that $\sqrt{7}$ is irrational.   (3 marks)

See next page

Question 13 **(8 marks)**

Let vector $\mathbf{a} = 6\mathbf{i} - 5\mathbf{j}$.

(a)   Determine the angle between $\mathbf{a}$ and $9\mathbf{i} + 8\mathbf{j}$.   (1 mark)

(b)   Let vector $\mathbf{b} = 21\mathbf{i} + t\mathbf{j}$. Determine the value of t so that $\mathbf{a}$ is

      (i)   parallel to $\mathbf{b}$.   (2 marks)

      (ii)   perpendicular to $\mathbf{b}$.   (2 marks)

(c)   Determine the vector projection of $\mathbf{a}$ on $-3\mathbf{i} + 4\mathbf{j}$.   (3 marks)

Question 14 **(8 marks)**

Two forces act on a body. F_1 has a magnitude of 285 N and acts on a bearing of 145. F_2 has a magnitude of 245 N and acts on a bearing of 075.

(a) Determine

(i) the magnitude and direction of the sum of the two forces. (6 marks)

(ii) the magnitude and direction of a third force that would keep the body in equilibrium. (1 mark)

(b)   The bearing F_2 acts on is changed so that the direction of $F_1 + F_2$ is due east. Determine the new bearing of F_2.   (4 marks)

Question 15 **(11 marks)**

Circle C has equation $(x + 3)^2 + (y - 5)^2 = 25$.

(a) Circle C is transformed by the matrix $M = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ to circle C'. Describe the direction and size of the rotation transformation M and state the equation of circle C'. (3 marks)
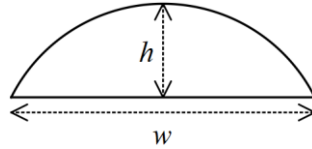
(b) Circle C' is then transformed by the matrix $N = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ to circle C''. Describe transformation N and state the equation of circle C''. (3 marks)

(c) Determine the single matrix P that will transform circle C'' back to circle C. (2 marks)

Question 16   **(6 marks)**

A segment of a circle has a perpendicular height of h and width w.

(a)   Determine the radius of the arc of the segment when $h = 4$ cm and $w = 10$ cm.   (3 marks)

(b)   Use the intersecting chord theorem to derive a formula for the radius of the arc of a segment of width w and height h, where the chords are the straight edge of the segment and the diameter of the circle.   (3 marks)

See next page

Question 17 **(7 marks)**

Let $N = \{0, 1, 2, 3, 4, 5\}$.

(a) Two or four-digit codes are to be formed using integers selected from N, such as 52 or 0307.

Determine the number of codes that can be formed if

(i) there are no restrictions. (2 marks)

(ii) no integer may be used more than once in a code. (2 marks)

(b) Using the pigeon-hole principle or otherwise, prove that when four integers are selected from N, at least one pair of the integers will have a sum of 5. (3 marks)

Question 18   **(8 marks)**

A small body P moves in a straight line. The displacement of the body from a fixed point O is given by $x = a\sin\big(b(t+c)\big) + d$, where x is in centimetres, t is the time in seconds. The graph of x against t is shown below.
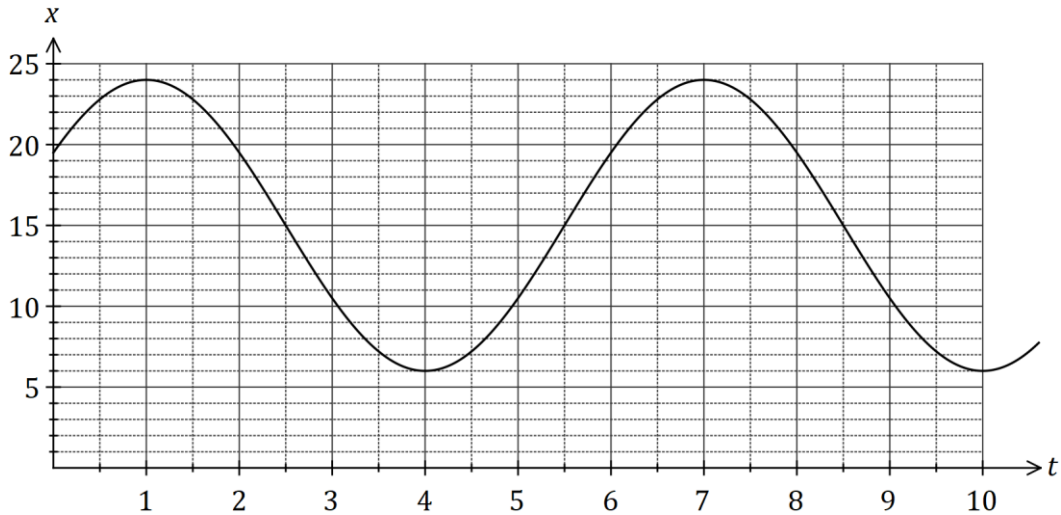
(a)   Determine the values of the **positive** constants a, b, c and d.   (4 marks)

(b)   Determine the first time that P is 18 cm from O after 150 seconds, giving your answer to two decimal places.   (2 marks)

(c)   Express the relationship between x and t as a cosine function.   (2 marks)

See next page

Question 19   **(6 marks)**

Use mathematical induction to prove that for all positive integers n

$$1 \times 7 + 2 \times 8 + 3 \times 9 + \cdots + n(n+6) = \frac{n}{6}(n+1)(2n+19).$$

Question 20   **(8 marks)**

(a)   The four points K, L, M and N lie in that order on the circumference of a circle. Chords KM and LN intersect at X. Prove that $\Delta KXL \sim \Delta NXM$.   (4 marks)

(b)   In triangle ABC, P, Q and R are the midpoints of AB, AC and BC respectively. If $\overrightarrow{AB} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$, use a vector method to prove that $PBRQ$ is a parallelogram.   (4 marks)

Question 21 **(7 marks)**

A small drone is to fly in a straight line and at a constant altitude from P to Q. Q lies 975 m away from P on a bearing of $155°$ and a steady wind of 5.6 ms^{-1} is blowing in the area from due east.

If the speed of the drone is set to 16.6 ms^{-1}, determine the bearing it should steer and the time that it will take to reach Q.

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

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